Coarsening of granular segregation patterns in quasi-two-dimensional tumblers

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A fundamental characteristic of granular flows is segregation on the basis of particle size or density. For bidisperse mixtures of particles, revolutions of the order of 10 produce a segregation pattern of several radial streaks in quasi-two-dimensional rotating tumblers with fill fractions between 50% and 70%. By extending the duration of the experiments to the order of 10^2–10^3 tumbler revolutions, we have found the first evidence of coarsening of the radial streak pattern to as few as one streak, resulting in an unexpected wedge-shaped segregation pattern. This phenomenon occurs for a wide range of conditions including several fill fractions, particle sizes and mixtures of particles varying in both size and density in circular tumblers as well as for particles varying in size in square tumblers. Coarsening seems to be driven by transport of small (or dense) particles from streak to streak through the semicircular radial core, leading to new questions about the physics of coarsening of granular segregation patterns.

Coarsening is a much studied subject in physics, but the coarsening of granular matter when subjected to flow is poorly understood. Theoretical understanding is lacking but, as we will show, even the experimental boundaries of what is possible—under what conditions does granular matter coarsen—are unclear as well.

Granular matter segregates during flow[1–7] on the basis of particle properties such as size and density. Examples are axial segregation in long tumblers[5,8–21] and the various types of segregation in quasi-two-dimensional (quasi-2D) tumblers, the simplest of these being 'classical' radial segregation[2–7], but also pattern formation in time-periodically rotating circular tumblers[22] and steadily rotating non-circular tumblers[23,24]. Of these, axial segregation stands out as one of the most studied and least understood granular phenomena. When a long cylinder with a circular or square cross-section is rotated about its axis, a mixture of large and small particles will separate into bands of mostly large particles and mostly small particles in $O(10^2)$ revolutions of the tumbler[5,8,9,13–17,21]. Over $O(10^3)$ tumbler revolutions, these bands merge or coarsen[5,10–13,16,18–20]. The details of how bands form and why coarsening occurs are not fully understood, but it is generally agreed that the process begins with radial segregation.

Classical radial segregation (a semicircular core of small particles surrounded by large particles) in steadily rotated quasi-2D circular tumblers is well understood. In a rotating tumbler with a surface flow in the continuous-flow or rolling regime[25], particles originally in the bed of particles in solid-body rotation with the tumbler enter the upstream portion of the flowing layer and fall out of the flowing layer further downstream. In the slightly dilated flowing layer, small particles percolate through the interstitial spaces of large particles. Size segregation results in small particles drifting towards the lower portion of the flowing layer and large particles drifting towards the upper portion of the flowing layer. As a result, the small particles quickly fall out of the flowing layer to occupy the inner radial core region in the bed of solid-body rotation, whereas the large particles fall out later to occupy the outer regions near the tumbler wall. Under reasonably general conditions, there is very good agreement between Poincaré sections derived from continuum models and experimental results for the cases of time-periodically rotating circular tumblers and steadily rotating non-circular tumblers[22–24]. In fact, a two-species model based on an inter-penetrating continuum—an unchanging underlying flow with segregation of the two species riding on top of this continuum—seems to capture the essential physics from a modelling viewpoint[3,23]. Roughly, the flow affects segregation, but segregation does not affect the flow.

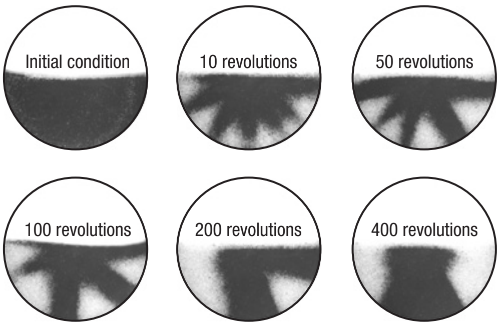

Figure 1 Radial streak coarsening in a bidisperse size-varying mixture. Images of a 55%-full quasi-2D circular tumbler rotated at 2 revolutions per minute (r.p.m.). The mixture is 50/50 by volume 1 mm painted black glass particles and 3 mm clear glass particles. The initially homogeneous mixture segregates into radial streaks in 10 revolutions. These streaks coarsen into one over several hundred revolutions.

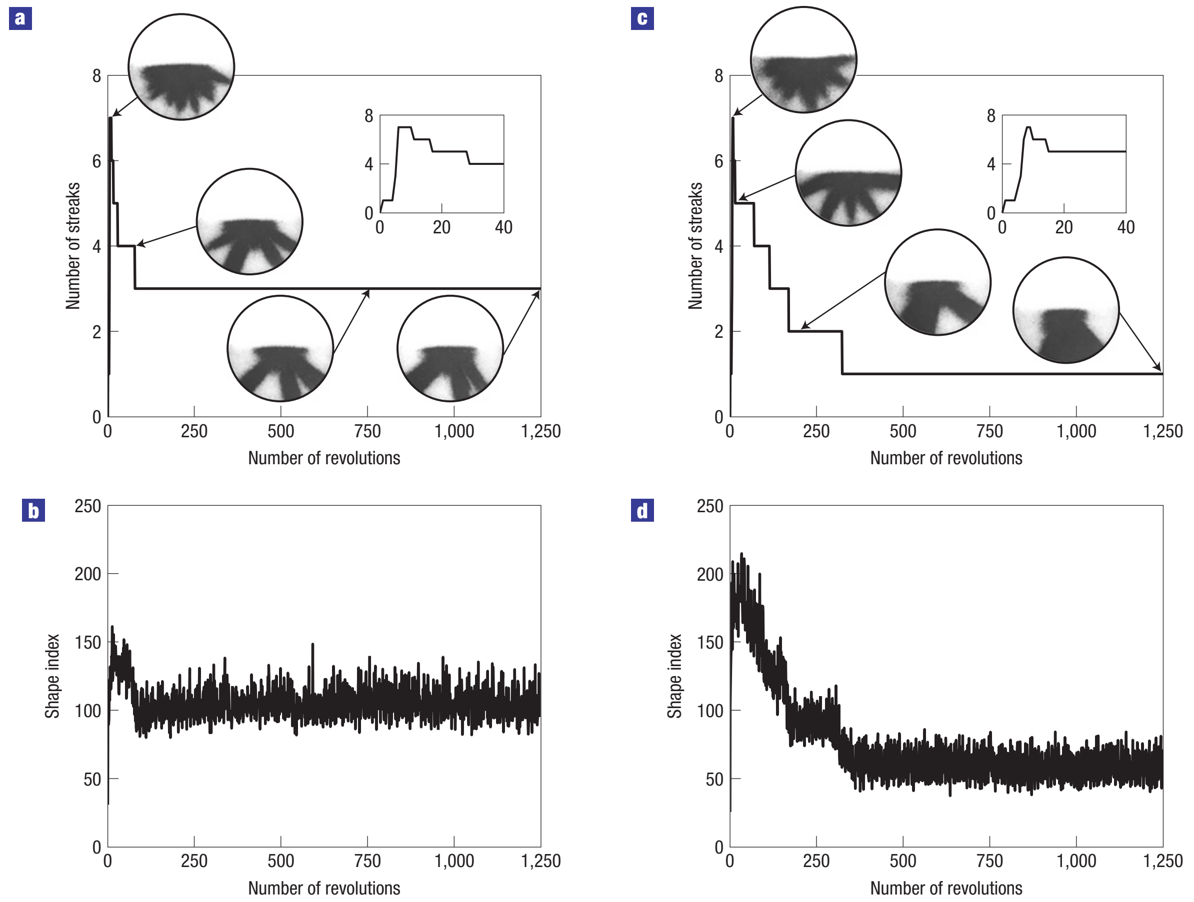

Figure 2 Evolution of streak coarsening in a size-varying mixture. The number of streaks and shape index versus tumbler revolutions for a size-varying mixture of 50/50 by volume 1 mm painted black glass particles and 3 mm clear glass particles in a quasi-2D circular tumbler. **a**, Number of streaks at 1 r.p.m. (inset shows results for the first 40 revolutions). **b**, Shape index at 1 r.p.m. **c**, Number of streaks at 2 r.p.m. (inset shows results for the first 40 revolutions). **d**, Shape index at 2 r.p.m.

But, as we shall show, segregation in quasi-2D is still imperfectly understood. When the system is operated in such a way that the assumption of unchanging underlying flow is violated, there is a coupling between flow and segregation resulting in a different segregation pattern: streaks form. Flow coupling occurs in rotating tumblers when the fill fraction is slightly more than half full. The 'sweet spot' occurs when a circular tumbler is 55% full[3,26,27]; this results in conditions that magnify slight differences in the flow of two species in the layer and leads to the presence of several streaks in the segregated pattern. In fact, for fill fractions between 50% and 70% on the basis of volume, a radial streak pattern may form[3,26–28] such as that shown in Fig. 1. Radial streaks can also occur for size-varying mixtures in square tumblers[3] or in circular or square tumblers when size and density effects reinforce each other in a bidisperse mixture (the small particles are heavy and the large particles are light). The result may be either a classical radial core or a radial streak pattern depending on the relative sizes and densities of the particles[29,30]. Radial petals have also been observed in avalanching flows of a size-varying mixture with a large difference in particle diameter in a half-full quasi-2D circular tumbler[31].

Like axial band formation in long cylindrical tumblers, the mechanism for the radial streak formation in quasi-2D tumblers is not clear. Hill *et al.* proposed that the streaks form in a continuously flowing layer owing to a wave-breaking mechanism[26,27]. Khakhar *et al.* used a continuum model with a piecewise-linear velocity profile owing to particle size differences and a moving interface to mimic variation in the dynamic angle of repose as different phases of particle types flow through the rapidly flowing surface layer[28]. Jain *et al.* proposed an argument based on mass balance for size- and density-varying systems[29,30]. As the flowing layer for large and light particles is thicker than that for small and heavy particles[32], the small and heavy particles flow faster to maintain mass balance. This higher fluidity in the streamwise direction results in radial streaks of small particles as they segregate in the tumbler.

SIZE-VARYING MIXTURES

Previous radial streaking studies have involved at most 45 tumbler revolutions[27], so no coarsening was observed, although streak merging and the generation of new streaks have previously been noted[28]. As shown for a bidisperse size-varying mixture in Fig. 1, an initially homogeneous mixture of large and small particles quickly segregates forming six radial streaks within 10 revolutions. These streaks gradually coarsen to as few as one streak over several hundred revolutions. The evolution of the coarsening of the size-varying mixture is illustrated by the number of streaks and the shape index, shown in Fig. 2. The shape index is the ratio of the perimeter of the segregation pattern squared to the area of the segregation pattern. The perimeter is found by calculating the distance between the centres of the pixels at the boundary between

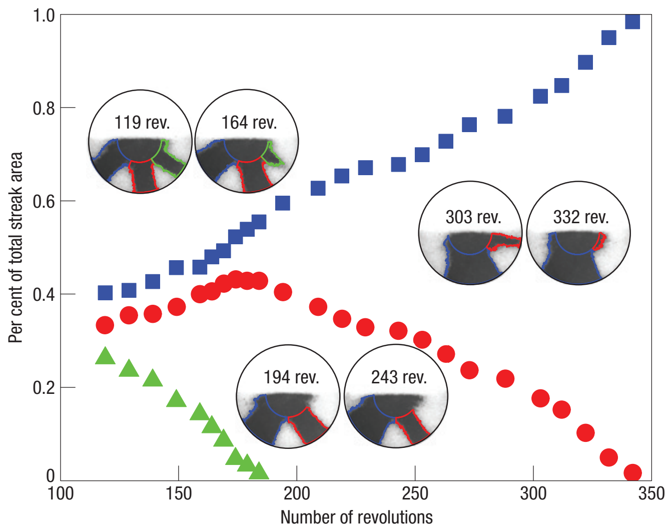

Figure 3 Dynamics of coarsening from three streaks to one in a size-varying mixture. After approximately 115 tumbler revolutions at 2 r.p.m. (Fig. 2c), the small particles have segregated into three large streaks. Over the next 65 tumbler revolutions, the left streak (area outlined in blue in the images; square symbols) and the middle streak (area outlined in red; circle symbols) gradually widen, whereas the right streak (area outlined in green in the images; triangle symbols) narrows. Over the next 170 tumbler revolutions, the streak outlined in red gradually narrows, whereas the streak outlined in blue gradually widens. The final steady state is one large streak.

the dark and light regions of the digital images that corresponds to the perimeter of the segregation pattern. The area of the segregation pattern is calculated from pixel counting. The shape index is at a maximum when there is a maximum number of streaks and can, therefore, be used as an automated means to detect the number of streaks and thereby confirm a visual count. The noisy shape index signal results from variations in the perimeter of the segregation pattern as the streaks of small particles pass through the flowing layer. At a tumbler rotation rate of 1 r.p.m. (Fig. 2a), the pattern quickly evolves to seven streaks (inset), but coarsens to four streaks within the first 30 revolutions. After 80 revolutions, it reaches a three-streak pattern, which is stable for the next 1,170 revolutions. At a tumbler rotation rate of 2 r.p.m. (Fig. 2c), the number of streaks grows to seven in nine revolutions (inset) but immediately coarsens, decreasing to five very quickly and then to two within the first 200 revolutions. Coarsening continues until only one streak persists after about 330 tumbler revolutions. The shape index (Fig. 2b,d) shows similar behaviour to the streak number, decreasing as the perimeter of the segregation pattern decreases. Repeating the experiment produces similar results, although details such as the number of tumbler revolutions at which streaks coarsen to a particular quantity vary from run to run.

The dynamics of the coarsening from three streaks to one streak on the basis of the measurement of the area of the streaks after subtracting the area of the semicircular core is illustrated in Fig. 3. After approximately 115 tumbler revolutions, there are three large streaks of the small particles. Over the next 65 tumbler revolutions, the right streak narrows and disappears, whereas the left and middle streaks widen. After approximately 180 tumbler revolutions, there are two large streaks of small particles. Over the next 170 tumbler revolutions, the streak on the right gradually narrows, whereas the streak on the left widens. The circumferential distance between the adjacent sides of the streaks does not change significantly as material is transferred between streaks, nor does the size of the semicircular core at the centre of the tumbler vary in size. Thus, there is an exchange of particles between the streaks through the radial core until there is finally one large streak.

SIZE- AND DENSITY-VARYING MIXTURES

Coarsening in quasi-2D tumblers is not limited to size-varying mixtures. Consider, for example, the size- and density-varying mixture at a tumbler rotation rate of 1 r.p.m., shown in Fig. 4a for 2,500 tumbler revolutions. Again within nine revolutions, the pattern reaches the maximum number of streaks, seven (inset), but coarsens to three streaks in 80 tumbler revolutions. This three-streak pattern is stable over the next 70 tumbler revolutions until a fourth streak re-emerges, disappears within 10 tumbler revolutions, and reappears for the next 170 tumbler revolutions. The pattern then coarsens briefly to two streaks, but a third streak quickly re-emerges. The three-streak pattern remains relatively stable, although the pattern coarsens briefly to two streaks.

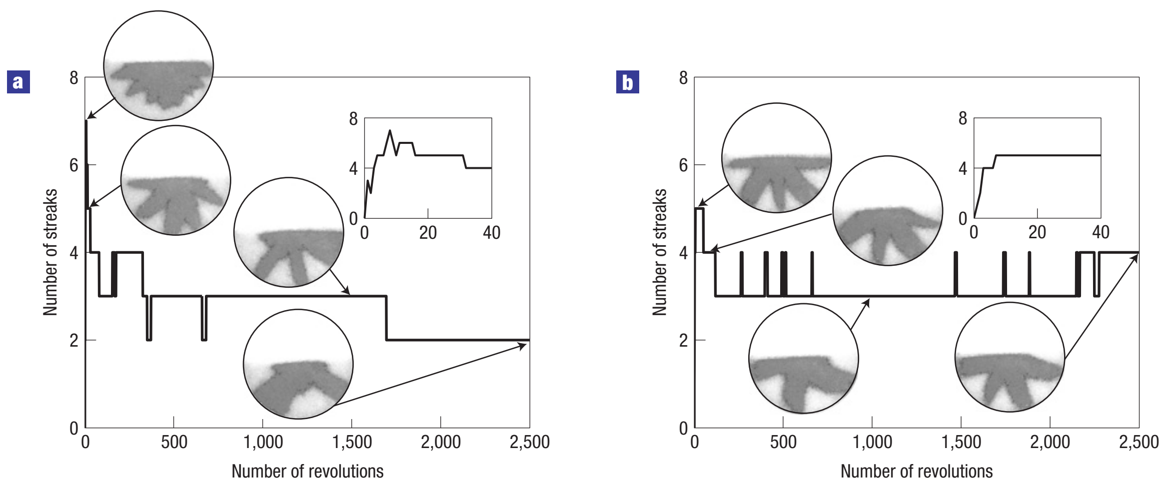

Figure 4 Coarsening for glass and steel particles of different sizes. The number of streaks versus number of tumbler revolutions for a size- and density-varying mixture of 50/50 by volume 1 mm steel particles and 3 mm clear glass particles in a quasi-2D circular tumbler. **a**, Number of streaks at 1 r.p.m. (inset shows results for the first 40 revolutions). **b**, Number of streaks at 2 r.p.m. (inset shows results for the first 40 revolutions).

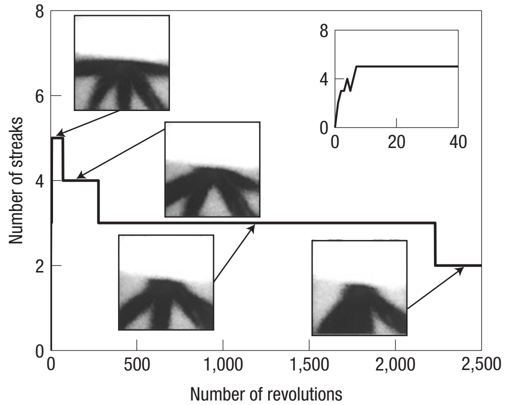

Figure 5 Coarsening in a square tumbler. The number of streaks (inset shows results for the first 40 revolutions) for a size-varying mixture of 50/50 by volume 1 mm painted black glass particles and 3 mm clear glass particles in a quasi-2D square tumbler rotated at 2 r.p.m.

Finally, the pattern coarsens to two streaks after about 1,700 tumbler revolutions.

In the size- and density-varying mixture at a tumbler rotation rate of 2 r.p.m., five streaks form within seven revolutions (Fig. 4b, inset), but the coarsening is slower and less effective than for the 1 r.p.m. case, as shown in Fig. 4b. The pattern coarsens to three streaks within 120 tumbler revolutions. The three streak pattern is predominant over the next 2,000 tumbler revolutions with a few instances of a fourth streak re-emerging. However, for the last 350 tumbler revolutions, a four-streak pattern dominates.

INFLUENCE OF SYSTEM PARAMETERS

Streak coarsening also occurs in a quasi-2D square tumbler, as shown for the size-varying mixture in a tumbler rotated at 2 r.p.m. in Fig. 5. The system reaches five streaks in seven revolutions (inset) that eventually coarsen to two after 2,230 revolutions. Streak coarsening was not observed within 2,500 revolutions at a rotation rate of 1 r.p.m.

The results presented here are a subset of a much larger set of experiments (over 60 different experiments) involving varying particle sizes from 0.3 mm to 3 mm, fill fractions between 50% and 70%, small particle concentrations of 20%, 40% and 50% by volume, tumbler diameters of 200 mm and 280 mm, and rotation rates between 1 r.p.m. and 4 r.p.m., all with a nearly flat continuously flowing surface layer. In all cases, streaks form rapidly, within ten tumbler revolutions. The general phenomenon of radial streak coarsening is reproducible, although the details as to the number of rotations when a given number of streaks coarsens vary from run to run. Coarsening is more likely to occur when radial streaks extend to the tumbler wall, as shown in figures Figs 1–5. When the streaks do not extend to the tumbler wall, coarsening is less likely.

MECHANISMS OF RADIAL STREAK COARSENING

Radial streak coarsening seems to be driven by the variation in the flow as each particle type traverses the layer[29,30]. For the size-varying mixture shown in Fig. 1, we have measured the velocities of the small and large particles at the free surface of the flowing layer using particle tracking velocimetry, a method described in detail elsewhere[33]. The small particles in the streaks flow 40% faster than the large particles from the regions between the streaks. Furthermore, the flowing layer for the small particles is approximately half the thickness of the flowing layer for large particles. Similar observations have been made for a size- and density-varying mixture[29].

Coupling the observations of the differences in surface velocity of the large (and light) and the small (and heavy) particles with the segregation mechanisms in the flowing layer may yield further understanding of the coarsening process. As they flow, small (heavy) particles sink towards the bottom of the flowing layer, pushing the large (light) particles towards the surface of the flowing layer[3]. In a pure streak of all small (heavy) particles, this segregation mechanism is not present. However, when an interface between the streaks of small (heavy) particles and the regions of large (light) particles enters the flowing layer, the particles initially remix and then re-segregate as they flow. It is likely that during this process a small fraction of the small (heavy) particles are re-distributed from the streak to the core of small (heavy) particles. This results in other particles from the core being redistributed into another streak.

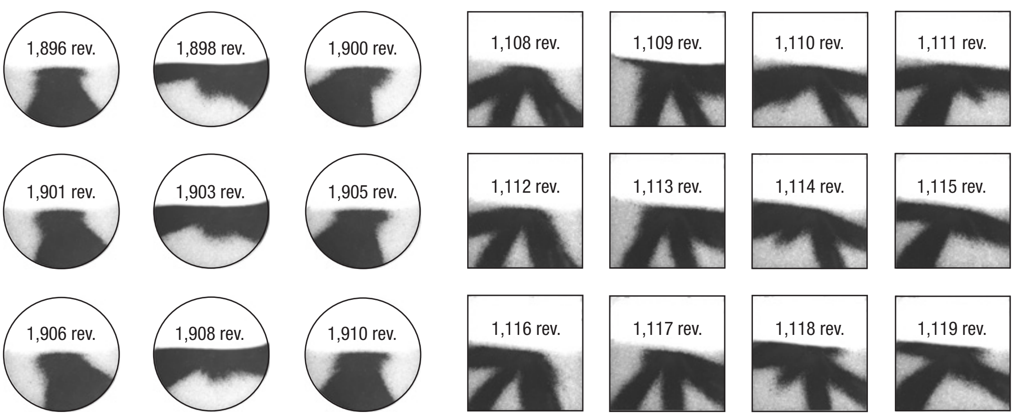

Figure 6 Pattern periodicity in circular and square tumblers. In the quasi-2D circular tumbler rotated at 2 r.p.m., the segregation pattern repeats its rotational orientation approximately every five tumbler revolutions (columns of images, left side). In the quasi-2D square tumbler rotated at 2 r.p.m., the segregation pattern repeats approximately every four tumbler revolutions (columns of images, right side). In both cases, the mixture is a 50/50 by volume size-varying mixture of 1 mm painted black glass particles and 3 mm clear glass particles.

Thus, the dynamics of segregation of particles at the interfaces of the streaks of small (heavy) particles and regions of large (light) particles could drive streak coarsening until the interfaces between the streaks of small particles and regions of large particles have been minimized.

ANALOGIES AND OPEN QUESTIONS

Streak coarsening in a quasi-2D tumbler has some analogies to coarsening of axial bands observed in long rotating tumblers where there exists a radial core[14,15,17,18] through which small particles are exchanged between bands leading to some bands decreasing in size and eventually disappearing, while the remaining bands grow[16,19–21]. Likewise, in quasi-2D tumblers, small and dense particles are exchanged between streaks via the semicircular radial core, causing some streaks to narrow while others widen, as illustrated in Fig. 3. Another similarity is the quick appearance of streaks followed by a slow coarsening process. For axial bands in long tumblers, the bands appear over $O(10^2)$ revolutions and coarsen over $O(10^3)$ revolutions, whereas radial streaks in quasi-2D tumblers form over $O(10)$ revolutions and coarsen over $O(10^2–10^3)$ revolutions.

An interesting aspect of the radial streak pattern is the periodicity of the rotational orientation of the streak pattern, shown in Fig. 6 for both the circular and square tumblers. When images of the segregation pattern are recorded once per revolution, the segregation pattern rotates relative to the tumbler. In the case of the quasi-2D circular tumbler rotated at 2 r.p.m., the period for the single-streak pattern to repeat is approximately five tumbler revolutions, whereas in the quasi-2D square tumbler rotated at 2 r.p.m., the period for the three-streak pattern to repeat is approximately four tumbler revolutions. The periodicity is apparently related to the ongoing percolation of small particles through the large particles in the flowing layer shifting the pattern relative to the tumbler. This observation may yield further insight into the quasi-2D coarsening process.

Perhaps even more interesting is that streak coarsening results in a wholly different segregation pattern than has ever been observed in a quasi-2D granular system. The semicircular segregation pattern is well known, and radial streaks or petals have been identified within the past few years[3,26–31]. However, the wedge-shaped pattern that occurs after coarsening is quite different from these previously identified segregation patterns. Of course, it is apparent that our results generate more questions than answers. The physics of the quasi-2D streak formation and coarsening and the physics of axial segregation into bands are undoubtedly different, and it is unlikely that one sheds much light on the other, although similarities exist.

METHODS

Particle mixtures include a 50/50 by volume bidisperse size-varying mixture of glass particles (1.19 ± 0.05 mm painted black and 3.03 ± 0.03 mm clear) and a 50/50 by volume bidisperse size- and density-varying mixture of small chrome steel particles (1.19 ± 0.03 mm) and large glass particles (3.03 ± 0.03 mm clear). The density of the painted black glass particles was $2.3\,\mathrm{g\,cm^{-3}}$, whereas the density of the clear glass particles was $2.4\,\mathrm{g\,cm^{-3}}$. The chrome steel particles had a density of $7.5\,\mathrm{g\,cm^{-3}}$. Both the size-varying and size- and density-varying mixtures were studied in a quasi-2D circular tumbler with a diameter of 280 mm and a thickness of 9 mm (approximately three large particle diameters). The size-varying mixture was also examined in a quasi-2D square tumbler with a side length of 254 mm and a thickness of 9 mm. The tumblers and end walls were made of clear acrylic. Static electricity effects were reduced through the use of an antistatic spray, which was allowed to dry before experiments began. Angular rotation, velocity and acceleration of the tumbler were computer-controlled via a stepper motor with an indexer. Digital images were taken once per revolution.

Both tumblers were initially filled with a homogeneous mixture of the two types of particle to a fill fraction of approximately 52%. However, once segregation occurred in just a few tumbler revolutions, both tumblers reached and maintained a fill fraction of $55 \pm 2\%$ owing to the less efficient packing of the segregated fractions and the slight dilation in the flowing layer. (The fill fraction did not change during the coarsening process.)

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Author contributions

S.W.M. conceived the experiments. S.W.M. and D.A.M.B. carried out the experimental work. All authors analysed the results and co-wrote the paper.