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The mathematical foundations of mixing. The linked twist map as a paradigm in applications. Micro to macro, fluids to solids. (English)

Cambridge Monographs on Applied and Computational Mathematics 22. Cambridge: Cambridge University Press. xix, 281 p. £ 40.00; $ 75.00 (2006). ISBN 0-521-86813-0/hbk

Mixing processes occur in many technological and natural applications with length and time scales ranging from the very small to the very large. At the starting point of any mixing process there are two or more constituents which occupy distinct domains whose size is on the order of the system size itself. The mixing process reduces the length scales of these materials below a certain level, resulting in a 'homogeneous' system – a mixture.

The diversity of problems connected with mixing in engineering, physics, chemistry, biology, oceanography, geophysics, astrophysics, etc., can give rise to a diversity of approaches. The monograph however takes the opposite approach. Its objective is on general results rather than on specifics. The central notion is that of mixing but of all the components of mixing the authors focus on only one – the kinematical aspects of mixing a fluid with itself. They study the motion due to an imposed velocity field, rather than study the hydrodynamic forces which give rise to the flows themselves.

The authors show how a variety of flows in very different settings possesses the characteristic of streamline crossing. This notion can be placed on firm mathematical footing via Linked Twist Maps (LTMs), which is the central organizing principle of this monograph. It is a rigorous mathematical book, most of the material presented is scattered throughout the mathematics literature. These original sources appeared in the pure dynamical systems community but the intended audience of the book under review consists of a broad spectrum of readers, from pure and applied mathematicians to engineers, physicists and geophysicists. Therefore, the authors decided to distil the original papers into a unified and user-friendly presentation of the idea of LTMs and to show how these results and the mathematical details within the now classical proofs relate to contemporary mixing problems. In my opinion, the authors succeed in doing this great job in an excellent way. Readers which are partially familiar with the topic will surely skip some pages. But even they will, regardless of whether they work in pure theory or in applications, read the rest with a great pleasure and as a reward will gain new insights into the topic.

I especially like the 'reader-friendly' style of the book. The authors provide help in navigating the book. The mathematical notions are of course defined rigorously but the authors add beautiful physical and heuristic explanations. All chapters have mini-introductions, a distillation of the contents of the chapters, and connections to fluid applications where possible. Each of the key Chapters 3-8 ends with a summary. Moreover, some key-points are summarized and written markedly in boxes at appropriate places throughout the text. Last but not least, there are 110 figures in the book.

The authors discuss the definition and construction of LTMs, provide examples of specific systems that can be analyzed in the LTM framework and introduce a number

of mathematical techniques which are then brought to bear on the problem of fluid mixing. They also present a number of open problems and new directions. The book consists of nine chapters: (1) Mixing: Physical issues, (2) Linked twist maps: definition, construction and the relevance to mixing, (3) The ergodic hierarchy, (4) Existence of a horseshoe for the linked twist map, (5) Hyperbolicity, (6) The ergodic partition for toral linked twist maps, (7) Ergodicity and the Bernoulli property for toral linked twist maps, (8) Linked twist maps on the plane, (9) Further directions and open problems. The references consist of 140 items.
I recommend the book to all scientists and graduate students interested in mixing. It is a pleasure to read it.

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Keywords : linked twist maps; mixing; ergodicity; Bernoulli property; horseshoe; Lyapunov exponent; Pesin theory; hyperbolicity; Arnold cat map; torus; plane
Classification :
∗37-02 Research exposition (Dynamical systems and ergodic theory)
37E40 Twist maps
37A25 Ergodicity, mixing, rates of mixing
37N10 Dynamical systems in fluid mechanics, oceanography and meteorology
37N25 Dynamical systems in biology
37B05 Transformations and group actions with special properties
37B10 Symbolic dynamics
37B25 Lyapunov functions and stability
37D20 Uniformly hyperbolic systems
37D25 Nonuniformly hyperbolic systems
76-02 Research monographs (fluid mechanics)